always energy-dependent preexponential factors which cannot generally be dismissed.

An extension of the present work to a three-dimensional setting is being considered.

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*Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. AFOSR-69-1727.

[1] The significance of this restriction is explained at the end of Sec. II.

[2] A careful discussion of the ordinary WKB approximation is given by N. Froman and P. O. Froman, *The J WKB Approximation* (North-Holland, Amsterdam, 1965).

[3] See also R. Bellman, *Perturbation Techniques in Mathematics, Physics and Engineering* (Holt, Rinehart, and Winston, New York, 1966), pp. 80–88.

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[5] One of the most thorough attempts to develop such an approach for heterojunctions and to provide a theoretical justification for the procedure is due to W. A. Harrison, Phys. Rev. 123, 85 (1961).

[6] These difficulties are exemplified by the controversy concerning the proper conditions to be imposed at points of discontinuity in the effective mass. [Compare, for instance, Harrison, Ref. 5 above, and J. W. Conley, C. W. Duke, G. D. Mahan, and J. J. Tiemann, Phys. Rev. 150, 466 (1966).]

[7] See E. L. Ince, *Ordinary Differential Equations* (Dover, New York, 1944), p. 122.

[8] Reference 2, p. 27.

[9] The characterization of the classical turning points at which the local velocity vanishes as singularities of $\epsilon(x)$ was noted by Froman. This point is further clarified by Eq. (3.1) below.

[10] See Ref. 3, p. 80.

[11] In principle, the turning points may be located well within the bulk region, far from the transition region. This however does not invalidate our assumption.

[12] Note that this bulk factor reduces to unity for a free particle.

[13] J. W. Conley and G. D. Mahan, Phys. Rev. 161, 681 (1967).

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PHYSICAL REVIEW B VOLUME 2, NUMBER 6 15 SEPTEMBER 1970

Ambipolar Transport of Electrons and Holes in Anisotropic Crystals*

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(Received 13 April 1970)

The problem of ambipolar diffusion and drift of electrons and holes, in which the diffusivity and mobility tensors for the two carrier species may be of different forms, is examined. Problems of this type arise, for example, in studying the diffusion and drift of excess carrier distributions in uniaxially stressed germanium and silicon as well as in certain naturally anisotropic substances. General ambipolar transport equations are obtained in situations where the quasineutrality approximation is justified. Solutions to these equations are quite easily obtained in certain cases where particular simplifying assumptions can be made. These solutions are explicitly obtained and the range of conditions under which they are applicable is outlined in detail. Certain other procedures have been employed to solve the general problem in cases when these conditions are not satisfied. Such methods are usually applicable only in cases involving rather special and restricted sample geometries. The transformation properties of the various terms in the ambipolar transport equation are discussed in various situations of physical interest and importance.

I. INTRODUCTION

The ambipolar transport behavior of excess carrier distributions in semiconductor crystals has been discussed by Herring,[1] Shockley,[2] and van Roosbroeck[3]; the most elegant and explicit treatment of the subject is contained in a subsequent article by the last author.[4] In this treatment the influences which the diffusing electrons and holes exert upon one another by virtue of their mutual electrostatic attraction are taken into account in an approximate way by assuming that the electrostatic forces are sufficient at all times to maintain an approximate state of electrical neutrality throughout the crystal. The electrons, whose mobility is greater, are thus envisioned as pulling the holes, whose mobility is less, along more rapidly than they would otherwise travel, and being themselves slowed down in the process. One may thus obtain values for the group diffusivity and group mobility of the excess carrier concentration distribution which are intermediate between the corresponding inherent electron and hole transport coefficients, and which moreover are, in general, concentration dependent. The requirement of electrical neutrality cannot, of course, be fulfilled precisely, for then the internal electric fields which arise to hold the diffusing distributions of opposite species together could never arise, but under a wide range of conditions (covering most circumstances which are experimentally realizable in germanium, silicon, and the III-V intermetallic semiconductors) the requisite fields may be set up by a very small fractional imbalance in electron and hole density.[5,6]

The predictions of this theory have been verified by experiment[7] and the theory itself is generally regarded as the basis for most of the subsequent studies related to the transport behavior of excess carriers or to the behavior of *p-n* junctions and semiconductor device structures. It has been extended in several directions, and these extensions have substantially increased the area of its applicability.[8–10] Until quite recently, all experimental studies involving the transport properties of excess carrier distributions in semiconductors were made with reference to isotropic samples. Recent studies[11,12] concerning the effects of uniaxial stresses upon minority carrier drift mobility in *p*-type germanium, however, have raised questions about the ambipolar diffusion and drift of excess carrier distributions in anisotropic crystals. Certain other specific instances involving the interaction of light with anisotropic crystals and the "pinch effect" which may arise when current flows through such crystals have also been considered in the literature.[13–16]

This problem would be a very straightforward one if the isotropy of electron and hole transport coefficients were affected in the same way when uniaxial stress is applied to a semiconductor crystal, but this is not ordinarily the case. Consider, for example, the case of germanium under uniaxial stress along the [111] direction, or silicon under uniaxial stress along [100]. In either instance, under compressive stresses, the ellipsoidal constant energy surface for electrons whose major axes are along the stress direction is lowered in energy, while the other surfaces belonging to the multivalley family are increased in energy. If sufficient compressive stress is brought to bear, all the electrons in the conduction band can be in the low-energy minima, and the transport properties of the conduction electrons in the crystal are then just those associated with a single ellipsoidal valley whose major axis is along the stress direction. Experimental studies have established the fact that in silicon and germanium, at any rate, the form of the ellipsoidal conduction-band surfaces are not strongly altered by the application of the stress.[17,18]

In the valence band, the constant energy surfaces *are* distorted to a considerable extent by the stress; by the application of sufficient compressive stress along certain important crystal directions, it has been shown[19,20] that the surfaces of constant energy become ellipsoids of revolution with the *stress direction* as the axis of revolution.[21] There is no direct relation between the magnitude of the anisotropy which arises in these rather different ways for electrons and for holes. In fact, for electrons the longitudinal effective mass is greater than the transverse mass, while for holes just the reverse is true. In the most general situation which might arise, the principal axes of the effective diffusivity and mobility tensors for electrons and holes may not coincide as they do in the examples discussed above. Behavior of this sort may be expected to occur in semiconductor crystals (such as tellurium) which are anisotropic in the absence of stress, when strains whose principal axes do not coincide with those of the zero-stress conductivity tensor are created.

In most of the investigations concerning ambipolar excess carrier transport which have been made to date[1–4,8–10] complete isotropy of the electron and hole transport coefficients has been assumed. It is the purpose of the present article, therefore, to extend the theory of ambipolar excess carrier transport in the most general way to situations wherein both electron and hole transport coefficients may be anisotropic such that the anisotropy associated with one carrier species may be altogether different from that of the other. The approach which will be taken will follow in a general way the one adopted by van Roosbroeck,[4] and the assumptions made by him in connection with the satisfaction of the quasineutrality conditions are made here also.

It is the fundamental purpose of this article to outline the circumstances under which ambipolar transport in anisotropic crystals can be treated as a simple extension of the isotropic case merely by introducing appropriate tensor diffusivities and mobilities, and when more fundamental effects arising from the crystal anisotropy are important and must also be considered.

II. TENSOR FORMULATION OF AMBIPOLAR TRANSPORT EQUATIONS

Under circumstances such as those outlined above, one must proceed by regarding the electron and hole mobilities as tensor quantities $\overleftrightarrow{\mu}_n$ (elements $\mu_n^{\alpha\beta}$) and $\overleftrightarrow{\mu}_p$ (elements $\mu_p^{\alpha\beta}$). The diffusivity tensors for electrons and holes $\overleftrightarrow{\mathrm{D}}_n$ (elements $D_n^{\alpha\beta}$) and $\overleftrightarrow{\mathrm{D}}_p$ (elements $D_p^{\alpha\beta}$) may then be expressed in terms of the mobility tensors by the Einstein relations[22]

$$\overleftrightarrow{\mathrm{D}}_n = (kT/e)\,\overleftrightarrow{\mu}_n \tag{1}$$

and

$$\overleftrightarrow{\mathrm{D}}_p = (kT/e)\,\overleftrightarrow{\mu}_p \;. \tag{2}$$

Maxwell-Boltzmann statistics are assumed to be applicable to both carrier species. The conductivity tensors $\overleftrightarrow{\sigma}_n$, $\overleftrightarrow{\sigma}_p$, and $\overleftrightarrow{\sigma}$, and the resistivity tensor $\overleftrightarrow{\rho}$ are defined by

$$\overleftrightarrow{\sigma}_p = pe\,\overleftrightarrow{\mu}_p \;, \tag{3}$$

$$\overleftrightarrow{\sigma}_n = ne\,\overleftrightarrow{\mu}_n \;, \tag{4}$$

$$\overleftrightarrow{\sigma} = \overleftrightarrow{\sigma}_p + \overleftrightarrow{\sigma}_n \;, \tag{5}$$

$$\overleftrightarrow{\rho} = \overleftrightarrow{\sigma}^{\,-1} \;, \tag{6}$$

where $n(x, y, z, t)$ and $p(x, y, z, t)$ are the electron and hole concentrations, respectively, at any point within the crystal at any time. We shall assume that the crystal is homogeneous and at the same temperature throughout.

The equations of continuity for electron and hole particle fluxes $\vec{\mathrm{J}}_n$ and $\vec{\mathrm{J}}_p$ are[4]

$$-\nabla\cdot\vec{\mathrm{J}}_n - \frac{n}{\tau_n} + g_0 + g' = \frac{\partial n}{\partial t} \;, \tag{7}$$

$$-\nabla\cdot\vec{\mathrm{J}}_p - \frac{p}{\tau_p} + g_0 + g' = \frac{\partial p}{\partial t} \;. \tag{8}$$

In these expressions τ_n and τ_p are the (concentration-dependent) electron and hole lifetimes, g_0 the equilibrium generation rate of electron-hole pairs, and g' the generation rate of electron-hole pairs in *excess* of equilibrium at any point (x, y, z) at time t. In this connection, one should note that since the total recombination rates for electrons and holes must be equal, we must have

$$n/\tau_n = p/\tau_p \;. \tag{9}$$

Likewise, the total generation rates of electrons and holes are equal since they must be created in pairs. For this reason, it is not necessary to distinguish between electron and hole generation rates in (7) and (8), and therefore the same symbols g_0 and g' are used for each.

The fluxes $\vec{\mathrm{J}}_n$ and $\vec{\mathrm{J}}_p$ may be expressed in terms of fields and concentration gradients as

$$\vec{\mathrm{J}}_n = \overleftrightarrow{\mathrm{D}}_n\cdot\nabla n - n\,\overleftrightarrow{\mu}_n\cdot\vec{\mathrm{E}} \;, \tag{10}$$

$$\vec{\mathrm{J}}_p = -\overleftrightarrow{\mathrm{D}}_p\cdot\nabla p + p\,\overleftrightarrow{\mu}_p\cdot\vec{\mathrm{E}} \;. \tag{11}$$

If the quansineutrality condition is satisfied one may equate excess electron and hole concentrations, whereby, letting n_0 and p_0 represent the concentration of electrons and holes in the equilibrium state,

$$p - p_0 = \delta p = \delta n = n - n_0 \tag{12}$$

and

$$\nabla p = \nabla(\delta p) = \nabla(\delta n) = \nabla n \tag{13}$$

Substituting (10) into (7) and (11) into (8), using (12) and (13) wherever possible to express quantities in terms of δp, one may easily obtain two equations for the excess carrier density having the form

$$\nabla\cdot[\overleftrightarrow{\mathrm{D}}_p\cdot\nabla(\delta p)] - \nabla\cdot(p\,\overleftrightarrow{\mu}_p\cdot\vec{\mathrm{E}}) + g' - \frac{\delta p}{\tau} = \frac{\partial(\delta p)}{\partial t} \tag{14}$$

and

$$\nabla\cdot[\overleftrightarrow{\mathrm{D}}_n\cdot\nabla(\delta p)] + \nabla\cdot(n\,\overleftrightarrow{\mu}_n\cdot\vec{\mathrm{E}}) + g' - \frac{\delta p}{\tau} = \frac{\partial(\delta p)}{\partial t} \;. \tag{15}$$

In these equations the quantity τ is an excess carrier lifetime defined by

$$\delta p/\tau = p/\tau_p - p_0/\tau_{p0} = n/\tau_n - n_0/\tau_{n0} \;, \tag{16}$$

where τ_{p0} and τ_{n0} represent the thermal equilibrium hole and electron lifetimes. The equality of the two defining equations follows from (9).

In the isotropic case $\overleftrightarrow{\mathrm{D}}_p$ and $\overleftrightarrow{\mu}_p$ are scalar quantities, and (14) and (15) then become

$$D_p\nabla^2(\delta p) - \mu_p\vec{\mathrm{E}}\cdot\nabla(\delta p) - \mu_p p\nabla\cdot\vec{\mathrm{E}} + g' - \frac{\delta p}{\tau} = \frac{\partial(\delta p)}{\partial t} \tag{17}$$

and

$$D_n\nabla^2(\delta p) + \mu_n\vec{\mathrm{E}}\cdot\nabla(\delta p) + \mu_n n\nabla\cdot\vec{\mathrm{E}} + g' - \frac{\delta p}{\tau} = \frac{\partial(\delta p)}{\partial t} \;. \tag{18}$$

One may now obtain an ambipolar transport equation by eliminating the term containing the divergence of $\vec{\mathrm{E}}$ between these two equations.[4] Alternatively, one may proceed by evaluating the divergence of $\vec{\mathrm{E}}$ from the expression

$$\vec{\mathrm{I}} = e(\vec{\mathrm{J}}_p - \vec{\mathrm{J}}_n) = \sigma\vec{\mathrm{E}} + e(D_n - D_p)\nabla(\delta p) \tag{19}$$

for the total current density, which follows directly from (10) and (11), and substituting the result into either (17) or (18), recalling, of course, that

$$\nabla\cdot\vec{\mathrm{I}} = 0 \;. \tag{20}$$

In either case, the result is found to be

$$D^*\nabla^2(\delta p) - \mu^*\vec{\mathrm{E}}\cdot\nabla(\delta p) + g' - \frac{\delta p}{\tau} = \frac{\partial(\delta p)}{\partial t} \;, \tag{21}$$

with

$$D^{*}=\frac{(n+p)D_nD_p}{nD_n+pD_p}\ , \quad \mu^{*}=\frac{(n-p)\mu_n\mu_p}{n\mu_n+p\mu_p}\ . \tag{22}$$

Equation (21) describes the transport of the excess carrier distribution δp in terms of an ambipolar diffusivity D^* and an ambipolar mobility μ^*. These coefficients are, of course, in general, concentration dependent, though in many frequently occurring instances (such as that of low injection level, where δp is everywhere much less than the majority carrier density) they are substantially constant.

In the anisotropic situation, unfortunately, it is impossible to eliminate completely the terms [corresponding to the $\nabla\cdot\vec{E}$ terms in (17) and (18)] containing spatial derivatives of the components of $\vec{E}$ between Eqs. (14) and (15). It is nevertheless instructive to proceed along somewhat similar lines by solving for the components of $\vec{E}$, substituting these into the transport equation, and utilizing the fact that $\nabla\cdot\vec{I}=0$. To do this let us first assume that our coordinate axes coincide with the principal axes of the diffusivity and mobility tensors of both electrons and holes. In doing this, of course, we exclude from our discussion cases where the principal axes of the transport tensors of the two carrier species do not coincide. In this reference system, then, we may write

$$\overleftrightarrow{D}_n=\begin{bmatrix} D_n^{11} & 0 & 0\\ 0 & D_n^{22} & 0\\ 0 & 0 & D_n^{33}\end{bmatrix}\ , \quad \overleftrightarrow{D}_p=\begin{bmatrix} D_p^{11} & 0 & 0\\ 0 & D_p^{22} & 0\\ 0 & 0 & D_p^{33}\end{bmatrix}\ , \tag{23}$$

while

$$\overleftrightarrow{\mu}_n=\begin{bmatrix} \mu_n^{11} & 0 & 0\\ 0 & \mu_n^{22} & 0\\ 0 & 0 & \mu_n^{33}\end{bmatrix}\ , \quad \overleftrightarrow{\mu}_p=\begin{bmatrix} \mu_p^{11} & 0 & 0\\ 0 & \mu_p^{22} & 0\\ 0 & 0 & \mu_p^{33}\end{bmatrix}\ . \tag{24}$$

The total current $\vec{I}$ [equal to $e(\vec{J}_p-\vec{J}_n)$] may, from (10) and (11), be expressed as

$$\vec{I}=\overleftrightarrow{\sigma}\cdot\vec{E}+e(\overleftrightarrow{D}_n-\overleftrightarrow{D}_p)\cdot\nabla(\delta p)\ . \tag{25}$$

This equation may be solved for the electric field components, the result being

$$E_\alpha=\frac{I_\alpha}{\sigma^{\alpha\alpha}}+\frac{e(D_p^{\alpha\alpha}-D_n^{\alpha\alpha})}{\sigma^{\alpha\alpha}}\frac{\partial(\delta p)}{\partial x_\alpha} \qquad \text{(no summation convention)}\ . \tag{26}$$

The partial derivatives

$$\frac{\partial E_\alpha}{\partial x_\alpha}=\frac{1}{\sigma^{\alpha\alpha}}\left(e(D_p^{\alpha\alpha}-D_n^{\alpha\alpha})\frac{\partial^2(\delta p)}{\partial x_\alpha^2}-e(\mu_p^{\alpha\alpha}+\mu_n^{\alpha\alpha})E_\alpha\frac{\partial(\delta p)}{\partial x_\alpha}+\frac{\partial I_\alpha}{\partial x_\alpha}\right) \tag{27}$$

may then be substituted into either (14) or (15), and using the condition that the divergence of the total current must vanish, that is,

$$\frac{\partial I_1}{\partial x_1}+\frac{\partial I_2}{\partial x_2}+\frac{\partial I_3}{\partial x_3}=0 \tag{28}$$

to eliminate $\partial I_3/\partial x_3$ from the resulting expression, one may finally obtain

$$\sum_\alpha D^{*\alpha\alpha}\frac{\partial^2(\delta p)}{\partial x_\alpha^2}-\sum_\alpha \mu^{*\alpha\alpha}E_\alpha\frac{\partial(\delta p)}{\partial x_\alpha}+\beta_1\frac{\partial J_1}{\partial x_1}+\beta_2\frac{\partial J_2}{\partial x_2}-\frac{\delta p}{\tau}+g'=\frac{\partial(\delta p)}{\partial t}\ , \tag{29}$$

where

$$D^{*\alpha\alpha}=(n+p)D_n^{\alpha\alpha}D_p^{\alpha\alpha}/(nD_n^{\alpha\alpha}+pD_p^{\alpha\alpha})\ , \tag{30}$$

$$\mu^{*\alpha\alpha}=(n-p)\mu_n^{\alpha\alpha}\mu_p^{\alpha\alpha}/(n\mu_n^{\alpha\alpha}+p\mu_p^{\alpha\alpha})\ , \tag{31}$$

$$\beta_1=(\sigma_n^{11}\sigma_p^{33}-\sigma_p^{11}\sigma_n^{33})/\sigma^{11}\sigma^{33}\ , \tag{32}$$

$$\beta_2=(\sigma_n^{22}\sigma_p^{33}-\sigma_p^{22}\sigma_n^{33})/\sigma^{22}\sigma^{33}\ , \tag{33}$$

$$J_\alpha=J_{p\alpha}-J_{n\alpha}\ . \tag{34}$$

This is the ambipolar transport equation for the excess carrier distribution δp expressed in the principal axis reference system. The ambipolar diffusivity tensor $\overleftrightarrow{D}^*$ and the ambipolar mobility tensor $\overleftrightarrow{\mu}^*$ assume a form which appears as a straightforward extension of van Roosbroeck's scalar expressions for these quantities in the isotropic case.[4] The terms involving the coefficients β_1 and β_2 are *purely anisotropic* in origin; these terms are absent in the isotropic theory and, indeed, it is evident from (32) and (33) that β_1 and β_2 vanish in an isotropic crystal.

To obtain solutions to (29) in the most general case, it is clearly necessary to know both $\vec{E}$ and $\vec{J}$. In the isotropic case, provided the quasineutrality condition is satisfied, it is possible to show that under a broad range of conditions, only the *applied* electric field need be considered explicitly in the ambipolar transport equation, the effects of any internal fields, arising from concentration gradients or other sources, being expressed implicitly through the modified ambipolar diffusivity and mobility coefficients.[23,24] If one may assume this procedure to be valid in the anisotropic case, then one might proceed initially by obtaining the applied field $\vec{E}$ from the solution of a purely electromagnetic boundary value problem. Once this quantity is known, the current density $\vec{J}$ and the excess carrier concentration δp may then be determined by solving (29) and (25) as a set of simultaneous equations in those variables. In general, of course, the equations are coupled, and it is extremely difficult to obtain any analytical solutions. If, however, the terms containing β_1 and β_2 may be neglected in comparison with the other terms in (29), these equations are uncoupled and analytic solutions can then be obtained by standard procedures. This will clearly be the situation in the isotropic case because then, of course, $\beta_1=\beta_2=0$, but even if the medium is anisotropic there are certain very important instances where this condition may still be satisfied. These cases may be enumerated as follows:

Case (i): low-level injection in extrinsic material. Clearly, for sufficiently small values of minority carrier conductivity the coefficients β_1 and β_2 will be so small as to render the effect of their respective terms in (29) negligible. This will ordinarily be the case when the majority carrier concentration is much greater than the intrinsic concentration, provided also that δp is everywhere small compared to the majority carrier density.

Case (ii): spatially constant current density. If the current density $\vec{I}$ vanishes, or is constant within the crystal, the derivatives $\partial J_1/\partial x_1$ and $\partial J_2/\partial x_2$ will be zero, and the effect of these terms will again be absent. This case is encountered in purely diffusive situations where applied fields are not present and external currents cannot flow. Note that this condition can be satisfied in some instances even with nonconstant current densities, since derivatives such as $\partial J_\alpha/\partial x_\beta (\alpha \neq \beta)$ need not necessarily be zero.

Case (iii): equal mobility ratios along three principal axes. The coefficients β_1 and β_2 will be zero if $\sigma_n^{11}\sigma_p^{33}=\sigma_p^{11}\sigma_n^{33}$ and $\sigma_n^{22}\sigma_p^{33}=\sigma_p^{22}\sigma_n^{33}$. In view of the definitions of $\overleftrightarrow{\sigma}_n$ and $\overleftrightarrow{\sigma}_p$ as expressed by (3) and (4), this will occur when $\mu_n^{11}/\mu_p^{11}=\mu_n^{22}/\mu_p^{22}=\mu_n^{33}/\mu_p^{33}$.

It is important to understand that the peculiar electrical and optical effects, such as the pinch effect[14–16] and the photopiezoresistance effect[13] which arise from the transport of carriers in anisotropic crystals, are always closely related to the terms containing β_1 and β_2 in (29) and are always absent in situations where those terms are negligible.

As a specific example of the behavior which is generally encountered in anisotropic crystals, consider a uniform crystal in which the conditions outlined above in connection with case i are satisfied, and assume that a δ-function pulse of excess carriers is injected at the origin at time $t=0$. A constant applied electric field $\vec{E}$ is assumed to be present. Then, the terms involving β_1 and β_2 in (29) are negligible, and one may proceed to show, by standard techniques, that the excess carrier density is given by

$$\delta p(x_1, x_2, x_3, t)=\frac{A}{(4\pi D_m^{11}D_m^{22}D_m^{33}t^3)^{1/2}} \times \exp\left(\sum_\alpha -\frac{(x_\alpha-\mu_m^{\alpha\alpha}E_\alpha t)^2}{4D_m^{\alpha\alpha}t}-\frac{t}{\tau}\right) \quad . \quad (35)$$

In this equation A is a constant and $D_m^{\alpha\alpha}$ and $\mu_m^{\alpha\alpha}$ refer to the *minority* carrier diffusivity and mobility. It is clear, of course, from (30) and (31) that in extrinsic samples the ambipolar coefficients $D^{*\alpha\alpha}$ and $\mu^{*\alpha\alpha}$ reduce to the respective minority carrier coefficients.

The surfaces of constant excess concentration associated with (34) are ellipsoids whose axes coincide with the principal axes of the diffusion-mobility tensors. The distribution expands and spreads in time as described by (34) and the center of the distribution moves with a constant group velocity $\vec{v}$ given by

$$\vec{v}=\sum_\alpha \mu_m^{\alpha\alpha}E_\alpha \vec{i}_\alpha \quad , \qquad (36)$$

where $\vec{i}_\alpha$ is a unit vector along the x_α axis. These results can be verified by direct substitution into the transport equation, and comparison with the specified boundary conditions. If the excess carrier density is so large that appreciable conductivity modulation is produced, or if the crystal is nonextrinsic to the extent that the values of the ambipolar transport coefficients depart significantly from those of the minority carrier transport coefficients, the terms in β_1 and β_2 in the transport equation become important. The surfaces of constant concentration are then *no longer* ellipsoidal, and the drift velocity $\vec{v}$ is *no longer* given by (36). Electrical effects such as the pinch effect become important, and the whole problem of finding the excess carrier concentration at any point becomes very complex.

Certain investigators have been able to obtain solutions to the ambipolar transport equation in restricted instances where none of the conditions set forth above are satisfied.[13–16] These solutions have been used to describe certain physical effects which fundamentally arise from the crystal anisotropy. Such solutions can be obtained only in very specific instances referring to highly restricted sample geometries and orientations where, for example, an expression for the current can be obtained by inspection and used to reduce the problem to one involving essentially only one-dimensional transport.

It will be noted that the procedure used above in eliminating $\partial I_3/\partial x_3$ by using Eq. (28) is arbitrary inasmuch as $\partial I_1/\partial x_1$ or $\partial I_2/\partial x_2$ could equally well have been eliminated from the transport equation. This is indeed the case, and correspondingly, Eq.

(29) can be written in other equally valid forms by eliminating those terms instead. The conclusions to be drawn in regard to the conditions under which the terms containing $\partial J_\alpha/\partial x_\alpha$ may safely be neglected are, nevertheless, the same in all cases. It might also be contended that it would have been more elegant to have rejected the initial choice of a specific coordinate system in favor of a general dyadic notation or at any rate a more general component representation. It is, indeed, possible to proceed using either of these alternative approaches, but the explicit form of the coefficients appearing in the terms expressing effects arising from crystal anisotropy is unclear when this is attempted. It must be remembered, however, that in Eq. (29) or any other alternative expression derived with reference to a particular coordinate system, that the transformation properties of the individual terms cannot be taken for granted. Thus, in (29), it is evident that the components $D^{*\alpha\alpha}$ and $\mu^{*\alpha\alpha}$ transform to other coordinate systems like the components of tensors, in view of the fact that they arise directly from the components of the tensors $\overleftrightarrow{D}_n$, $\overleftrightarrow{D}_p$, $\overleftrightarrow{\mu}_n$, and $\overleftrightarrow{\mu}_p$. It is not, however, to be expected that the transformation properties of β_1 and β_2 are equally simple. Their transformation properties can be ascertained from the results of similar calculations using generalized coordinate systems, but are quite complicated, in general, and will not be discussed here. In using Eq. (29), then, other coordinate systems can be adopted, provided that the conditions outlined in cases (i), (ii), or (iii) above are satisfied, and that a proper tensor transformation of the components $D^{*\alpha\alpha}$ and $\mu^{*\alpha\alpha}$ is made. If those conditions are not satisfied, then the terms containing β_1 and β_2 are important and one must restrict oneself to the initially chosen principal axis system.

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*Work supported in part by the U. S. Air Force, Air Force Office of Scientific Research, under Grant No. AF-AFOSR-73-66.

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